Notes on the Helical Field

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The general form of the field in the current–free region of a helical magnet can be obtained by using the scalar potential ψ. A cylindrical coordinate system (r, ϕ, z) is more appropriate for this case because the helical symmetry is easily described in these coordinates. Helical symmetry demands that the physical system is invariant to the transformation $\phi - kz =$const. Thus ψ can be written as:

$$\psi = \psi\,(r, \theta)$$

where $\theta = \phi - kz$, and k is positive for a right–handed helix (if x, y, z is a right–handed Cartesian coordinate frame and $x = r\cos\phi$; $y = r\sin\phi$).

Solving Laplaces equation, the scalar potential is written as an infinite sum of harmonics:

$$\psi = \sum_{m=1}^{\infty} I_m\,(mkr) \cdot (a_m \cos\,(m\theta) + b_m \sin\,(m\theta)) \tag{1}$$

where I_m are modified Bessel functions and the coefficients a_m, b_m depend on the magnet coil configuration.

Then from the magnetic field equation $\mathbf{B} = -\text{grad}\ \phi$ one can immediately obtain:

$$\begin{aligned} B_r &= -k \sum_{m=1}^{\infty} m \cdot I'_m\,(mkr) \cdot (a_m \cos\,(m\theta) + b_m \sin\,(m\theta)) \\ B_z &= k \sum_{m=1}^{\infty} m \cdot I_m\,(mkr) \cdot (b_m \cos\,(m\theta) - a_m \sin\,(m\theta)) \\ B_\phi &= -\frac{1}{kr} B_z \end{aligned} \tag{2}$$

The helical symmetry results in the simple relation between B_ϕ and B_z. I'_m denotes the derivative on the argument of Bessel function (not on r).

The expression (1) for ψ is similar to the expression for the scalar potential of a normal magnet:

$$\Phi = \sum_{m=1}^{\infty} r^m \cdot (\alpha_m \cos(m\phi) + \beta_m \sin(m\phi)) \tag{3}$$

and at the consideration of a_m and b_m coefficients for helix one can go the same way as for normal magnet.

In particular if we consider a helical dipole and choose the on–axis field to be vertical at $z = 0$ then from dipole symmetry only harmonics with odd m are allowed and all a_m are equal to 0 (they are skew–harmonics). Further to eliminate harmonics with $m = 3, 5...$ one can use in the cross section of helix the same coil configuration which is used for eliminating higher order harmonics on normal dipoles.

From (2) one can see that the m–th harmonic term b_m contributes to nonlinearities of polynomial order $m-1$ and higher ($I_m(mkr)$ behaves like $(mkr/2)^m/m!$ when r goes to 0). So one would try to remove these harmonics/ But unlike a normal dipole the main harmonic of a helical dipole contains a nonlinear field. It is unavoidable. The b_1 coefficient of main harmonic is related with on–axis field B_0 as: $b_1 = -2B_0/k$. Thus the main harmonic of the field of a helical dipole can be written as:

$$\begin{aligned} B_r &= 2B_0 I_1'(kr) \cdot \sin\theta \\ B_z &= -2B_0 I_1(kr) \cdot \cos\theta \\ B_\phi &= -\frac{1}{kr} B_z \end{aligned} \tag{4}$$

Taking the limit $r \to 0$ one obtains the field near the axis of a helix. It is expressed in Cartesian coordinates as:

$$\begin{aligned} B_x &= -B_0 \cdot \sin(kz) \\ B_y &= B_0 \cdot \cos(kz) \\ B_z &= -B_0 k \cdot (x\cos(kz) + y\sin(kz)) \end{aligned}$$